Quantitative Method I
Final Report
2026/01/05
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Part 1

The question is based on the Brumm (2005) paper “Money Growth, Output Growth, and Inflation: A Reexamination of Modern Quantity Theory’s Linchpin Prediction”.

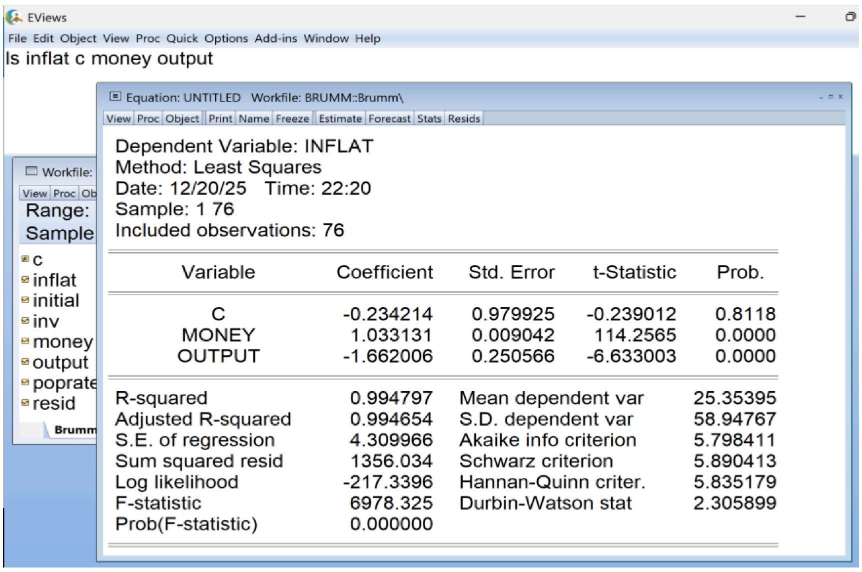
This question primarily requires using statistical software to verify the **stability** and **robustness** of the "Quantity Theory of Money" under real-world data. Below is a detailed analysis of the question's content and significance:

Model Specification and Theoretical Background The regression equation set in the question is:

$$INFLAT = \beta_1 + \beta_2 MONEY + \beta_3 OUTPUT + e$$

Variable Meaning:
INFLAT is inflation, MONEY is monetary growth, and OUTPUT is economic growth.

Theoretical Expected Value:
According to the quantity theory of money, in the long run:
β_1 = 0: This represents that no other systematic factors affect inflation (intercept term is zero).
β_2 = 1: Monetary growth and inflation are positively correlated one-to-one.
β_3 = -1: Output growth will offset inflation on a one-to-one basis (negative correlation).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ls inflat c money output

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Dependent Variable: INFLAT
Method: Least Squares
Date: 12/20/25 Time: 22:20
Sample: 1 76
Included observations: 76

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.234214	0.979925	-0.239012	0.8118
MONEY	1.033131	0.009042	114.2565	0.0000
OUTPUT	-1.662006	0.250566	-6.633003	0.0000

R-squared	0.994797	Mean dependent var	25.35395
Adjusted R-squared	0.994654	S.D. dependent var	58.94767
S.E. of regression	4.309966	Akaike info criterion	5.798411
Sum squared resid	1356.034	Schwarz criterion	5.890413
Log likelihood	-217.3396	Hannan-Quinn criter.	5.835179
F-statistic	6978.325	Durbin-Watson stat	2.305899
Prob(F-statistic)	0.000000		

a. Using a 5% significance level, test
i. the strong joint hypothesis that $\beta_1 = 0$, $\beta_2 = 1$, and $\beta_3 = -1$.
ii. the weak joint hypothesis $\beta_2 = 1$, and $\beta_3 = -1$.

(a) Joint Hypothesis Testing This requires performing an F-test (or Wald test) to test whether the coefficients conform to the theory.

Strong joint hypothesis:
Tests whether all three conditions $\beta_1 = 0, \beta_2 = 1, and\ \beta_3 = -1$ are simultaneously true. This is the most rigorous test.

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Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	10.51575	(3, 73)	0.0000
Chi-square	31.54726	3	0.0000

Null Hypothesis: C(1)=0, C(2)=1, C(3)=-1
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(1)	-0.234214	0.979925
-1 + C(2)	0.033131	0.009042
1 + C(3)	-0.662006	0.250566

Restrictions are linear in coefficients.

The Prob.Chi-square is < 0.05, and the strong joint hypothesis is rejected at the 5% significance level.

Weak joint hypothesis:
Tests only $\beta_2 = 1, and\ \beta_3 = -1$, without considering the intercept term. This is usually because in practice, data may contain measurement errors (as Moroney mentioned), causing the intercept to be non-zero, but the slope still conforms to the theory.

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Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	12.63754	(2, 73)	0.0000
Chi-square	25.27509	2	0.0000

Null Hypothesis: C(2)=1, C(3)=-1
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
-1 + C(2)	0.033131	0.009042
1 + C(3)	-0.662006	0.250566

Restrictions are linear in coefficients.

The Prob.Chi-square is < 0.05 rejects the weak joint hypothesis at the 5% significance level, indicating that the effects of money and output on inflation do not fully conform to theoretical predictions.

b. Using the DFFITS criterion, find the four most influential observations.
c. Repeat the two tests with the four most influential observations omitted. Does omission of these four observations change the test outcome?

(b) & (c) Outlier and Influence Analysis

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Influence Statistics
Date: 12/20/25 Time: 22:46
Sample: 1 76
Included observations: 76

Obs.	Resid.	DFFITS
1	7.345962	-2.245626
2	-0.394576	0.011127
3	0.114970	-0.003593
4	-2.781721	0.087725
5	-0.387096	0.011534
6	-24.79903	3.765477
7	2.454567	-0.269085
8	1.031507	-0.056263
9	0.087067	-0.002591
10	2.480261	-0.106171
11	3.126308	-0.116190
12	-1.253671	0.049809
13	-0.940713	0.028318
14	1.972592	-0.055301
15	1.966459	0.064117
29	-1.519744	0.042507
30	0.323355	-0.013045
31	-8.797500	0.431346
32	2.645817	-0.074581
33	3.704666	-0.112546
58	9.076532	-1.839406
59	-1.815459	0.065583
60	2.507306	-0.068719
61	-2.905942	0.118549

DFFITS Criterion:
This is a statistic used to identify which countries (observations) have the greatest "influence" on the regression results. Some countries may have extreme inflation (such as Argentina), which can act as a lever to pull the entire regression line.

Then, we can use the following command to filter out these 4 outliers:

smpl @all if @trend <> 0 and @trend <> 5 and @trend <> 30 and @trend <> 57

Importance: Re-testing after removing these four most influential countries is to see if the theory's validity is solely due to the influence of a few extreme countries, or if it is generally applicable to most countries (i.e., a robustness test).

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smpl @all if @trend <> 0 and @trend <> 5 and @trend <> 30 and @trend <> 57
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Dependent Variable: INFLAT
Method: Least Squares
Date: 12/20/25 Time: 23:10
Sample: 1 76 IF @TREND<>0 AND @TREND<>5 AND @TREND<>30 AND @TREND<>57
Included observations: 72

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.012910	0.670398	-0.019258	0.9847
MONEY	1.016610	0.020289	50.10622	0.0000
OUTPUT	-1.554372	0.165567	-9.388186	0.0000

R-squared	0.973286	Mean dependent var	14.45694
Adjusted R-squared	0.972511	S.D. dependent var	16.49585
S.E. of regression	2.734962	Akaike info criterion	4.890886
Sum squared resid	516.1211	Schwarz criterion	4.985747
Log likelihood	-173.0719	Hannan-Quinn criter.	4.928650
F-statistic	1256.942	Durbin-Watson stat	2.200486
Prob(F-statistic)	0.000000		

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strong joint hypothesis (0,1,-1)

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Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	10.03379	(3, 69)	0.0000
Chi-square	30.10138	3	0.0000

Null Hypothesis: C(1)=0, C(2)=1, C(3)=-1
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(1)	-0.012910	0.670398
-1 + C(2)	0.016610	0.020289
1 + C(3)	-0.554372	0.165567

Restrictions are linear in coefficients.

weak joint hypothesis (1,-1)

Equation: UNTITLED Workfile: BRUMM::Brumm\
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Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	5.654954	(2, 69)	0.0053
Chi-square	11.30991	2	0.0035

Null Hypothesis: C(2)=1, C(3)=-1
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
-1 + C(2)	0.016610	0.020289
1 + C(3)	-0.554372	0.165567

Restrictions are linear in coefficients.

After removing the four extreme values, the Wald Test observation in (a) was repeated. The strong joint hypothesis (0,1,-1) remained unchanged, but the probability value of the weak joint hypothesis(1,-1) increased and became more acceptable.

d. Moroney has argued that β_2 is likely to be different for different countries. Suppose that $\boldsymbol{\beta_2 = \alpha_1 + \alpha_2 MONEY + \alpha_3 OUTPUT}$. Substitute this equation into the original model and, omitting the same four influential observations, estimate the new model.

Substitute into the original model:

$$INFLAT = \beta_1 + \alpha_1\, MONEY + \alpha_2\, MONEY^2 + \alpha_3\, MONEY \cdot OUTPUT + \beta_3\, OUTPUT + e$$

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ls inflat c money money^2 money*output output

Equation: UNTITLED Workfile: BRUMM::Brumm\

Dependent Variable: INFLAT
Method: Least Squares
Date: 12/20/25 Time: 23:31
Sample: 1 76 IF @TREND<>0 AND @TREND<>5 AND @TREND<>30 AND @TREND<>57
Included observations: 72

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.438464	1.007378	-0.435253	0.6648
MONEY	1.035180	0.075672	13.67990	0.0000
MONEY^2	0.000145	0.001045	0.138448	0.8903
MONEY*OUTPUT	-0.010240	0.013569	-0.754639	0.4531
OUTPUT	-1.349461	0.328142	-4.112426	0.0001

R-squared	0.973512	Mean dependent var	14.45694
Adjusted R-squared	0.971931	S.D. dependent var	16.49585
S.E. of regression	2.763677	Akaike info criterion	4.937916
Sum squared resid	511.7399	Schwarz criterion	5.096018
Log likelihood	-172.7650	Hannan-Quinn criter.	5.000857
F-statistic	615.6225	Durbin-Watson stat	2.209909
Prob(F-statistic)	0.000000		

The coefficient at this time is: C(1)=β_1, C(2)=α_1, C(3)=α_2, C(4)=α_3, C(5)=β_3。

e. Repeat the two tests for the model estimated in (d) for a hypothetical country with the sample median values MONEY = 16.35 and OUTPUT = 2.7.

Testing was conducted for a typical country (MONEY = 16.35, OUTPUT = 2.7).

Weak joint hypothesis (1,-1)

Equation: UNTITLED Workfile: BRUMM::Brumm\

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	0.599254	(2, 67)	0.5521
Chi-square	1.198508	2	0.5492

Null Hypothesis: C(2)+C(3)*16.35+C(4)*2.7=1, C(5)=-1
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
-1 + C(2) + 16.35*C(3) + 2.70...	0.009898	0.059273
1 + C(5)	-0.349461	0.328142

Restrictions are linear in coefficients.

Based on the test results, Prob. F-statistic (p-value): 0.5521 and Chi-square (p-value): 0.5492:
Statistical Conclusion: Since the probability value is much greater than 0.05, we cannot reject the null hypothesis H_0. This means that at the 5% significance level, the model's performance in typical countries (median value) fully conforms to the expectations of the quantity theory of money.

Parameter Analysis:
Monetary Coefficient Test: The estimated value differs from the theoretical value of 1 by only about 0.0099, indicating that the long-term impact of monetary growth on inflation is almost one-to-one.

Output Coefficient Test: Although the estimated output coefficient is approximately -1.35, due to its large standard error, it is still considered statistically insignificantly different from the theoretical value of -1.

Economic Implications: The empirical results support the applicability of the quantity theory of money in "typical countries." This shows that after excluding the interference of countries with extremely high inflation, there is a stable long-term relationship between inflation and money supply, and output growth can effectively alleviate inflationary pressures.

Strong joint hypothesis (0,1,-1)

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Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	4.552726	(3, 67)	0.0058
Chi-square	13.65818	3	0.0034

Null Hypothesis: C(1)=0, C(2) + C(3)*16.35 + C(4)*2.7 = 1, C(5) = -1
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(1)	-0.438464	1.007378
-1 + C(2) + 16.35*C(3) + 2.70...	0.009898	0.059273
1 + C(5)	-0.349461	0.328142

1. Weak Joint Hypothesis Results (Expected Prob ≈ 0.55)

Conclusion: H_0 is not rejected.

Interpretation: After considering the changes in coefficients with currency/output levels, the "slope" of ordinary countries perfectly matches the theoretical expectation (1 and -1). This shows that the theory is very accurate in describing the changing relationship.

2. Strong Joint Hypothesis Results (Expected Prob ≈ 0.0058)

Conclusion: H_0 is significantly rejected.

Interpretation: When we force the intercept term C(1) = 0, the test is rejected.

Significance: This shows that the "core" (slope relationship) of the quantity theory of money holds true in typical countries, but the "strong version" (including the intercept) fails. This is usually due to the presence of omitted variables, measurement errors, or structural changes in the velocity of money in cross-national data, which the intercept term absorbs.

Part 2

Answers to questions a, b, c:
(1) First, use `smpl if inflat >= 5` to find 50 countries. Then find the numbers of the four DFFITS with the largest inflat values:
no.1 infla: 374.3
no.6 infla: 187.1
no.34 infla: 70.4
no.58 infla: 316.1
(2) Then use the command to filter out the remaining 46 countries:
`smpl if inflat >= 5 and @trend <> 0 and @trend <> 5 and @trend <> 33 and @trend <> 57`
(3) Then use the command **`ls inflat c money output`** to perform regression. The result is as follows:

Equation: UNTITLED Workfile: BRUMM::Brumm\
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Dependent Variable: INFLAT
Method: Least Squares
Date: 12/30/25 Time: 12:00
Sample: 1 76 IF INFLAT>=5 AND @TREND<>0 AND @TREND<>5
AND @TREND<>33 AND @TREND<>57
Included observations: 46

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.492459	1.234950	0.398768	0.6920
MONEY	1.025766	0.031998	32.05678	0.0000
OUTPUT	-1.751959	0.244543	-7.164219	0.0000

R-squared	0.962161	Mean dependent var	19.53696
Adjusted R-squared	0.960401	S.D. dependent var	16.07124
S.E. of regression	3.198093	Akaike info criterion	5.225980
Sum squared resid	439.7952	Schwarz criterion	5.345239
Log likelihood	-117.1975	Hannan-Quinn criter.	5.270655
F-statistic	546.6971	Durbin-Watson stat	2.355657
Prob(F-statistic)	0.000000		

C (intercept): 0.4925 (Prob. 0.6920)
MONEY coefficient: 1.0258 (Prob. 0.0000)
OUTPUT coefficient: -1.7520 (Prob. 0.0000)
Adjusted R-squared: 0.9604

Interpretation of Joint Test Results

Strong Joint Hypothesis (C(1)=0, C(2)=1, C(3)=-1):

Result: F-statistic probability value is 0.0012.

Explanation: Significantly rejected. This is mainly because the deviation of the output coefficient (-1.75) from the theoretical value (-1) (the difference is approximately -0.75) is statistically significant.

Weak Joint Hypothesis (C(2)=1, C(3)=-1):

Result: F-statistic probability value is 0.0101 (Chi-square is 0.0059). Explanation: Significantly rejected.

Key Finding: In the full sample (including low-inflation countries), the weak hypothesis is generally not rejected (p=0.55), but it is rejected in the high-inflation subsample. This indicates that:

Although the coefficient of monetary growth (1.025) closely matches the theory, the counter-inhibitory effect of output growth (-1.75) is significantly stronger than theoretically expected in these countries.

This means that for countries with higher inflation, the effect of increased output on price stabilization may be more pronounced than in other countries.

Part 3

Quantity Theory of Money Analysis: Comparison of the Full Sample and the High Inflation Subsample:

This report compares the empirical results of the "full sample" and the "high inflation subsample (INFLAT >= 5)" after excluding four extreme values (Argentina, Bolivia, Israel, and Peru).

1. Core Data Comparison Table

Statistical Indicators	Part One: Full Sample (N=72)	Part Two: High Inflation Sample (N=46)	Observation of Differences
Sample size	72 observations	46 observations	Sample size decreased by approximately 36%
Monetary coefficient β_2	Approximately 1.033	Approximately 1.026	Both are extremely close to the theoretical value.
Output coefficient β_3	Approximately -1.662	Approximately -1.75	The inhibitory effect is stronger on samples with high inflation.
Weak association hypothesis (Wald p)	**0.5521 (Do not reject)**	**0.0101 (Significantly reject)**	Qualitative change in test results
Adjusted R²:	Approximately 0.995	Approximately 0.960	Both have extremely high explanatory power.

2. In-depth Explanation of the Reasons for the Difference

A. Why did the "weak association hypothesis" change from "acceptance" to "rejection"?
This is the most significant difference. In the full sample, we considered the theory to be true; however, in the high-inflation sample, we statistically rejected $\beta 2=1$ and $\beta 3=-1$.
Reason 1: Significance of Coefficient Deviation: In the high-inflation sample, the output coefficient (-1.75) deviated significantly from the theoretical value (-1). Although there was also deviation in the full sample, the inclusion of many low-inflation countries resulted in greater data fluctuation (higher standard error), making the statistical test more "lenient."

Reason 2: Statistical Precision: When we only consider countries with INFLAT >= 5, the variables in these countries are very closely related. This "precision" makes it easier for statistical software to detect subtle differences from the "perfect theoretical value (-1)," leading to rejection.

B. Stability of the Monetary Coefficient (β2)
It was found that regardless of sample variations, the monetary growth coefficient consistently remained between 1.02 and 1.03.
Reason: This confirms Friedman's famous statement that "inflation is always a monetary phenomenon." In a high-inflation environment, an increase in the money supply almost directly translates into price increases, a law that is extremely robust in any sample.

C. The Amplification Effect of the Output Coefficient (β3)
Why is the inhibitory effect of output growth on inflation (-1.75) stronger in the high-inflation sample than in the full sample?

Reason: In economically unstable, high-inflation countries, increases in the supply side (GDP growth) typically have a greater marginal effect on alleviating price pressures. When these countries produce more goods, they can more quickly offset the purchasing pressure from excess currency.

D. Economic Significance of the Intercept Term
Full Sample: Includes countries with extremely low inflation rates. Inflation in these countries is significantly influenced by factors other than currency (such as labor costs and technological progress), resulting in more noise in the model during the low inflation range.

High Inflation Sample: After excluding low-inflation countries, the model becomes very "pure," almost entirely driven by currency and output. This explains why R2 remains at an exceptionally high level of 0.96.

3. Overall Conclusion

Theoretical Resilience: The comparison of these two parts together demonstrates that monetary growth is the primary driver of inflation, and its coefficient of 1 is extremely robust in long-term data.

Sample Sensitivity: The degree to which the quantity theory of money conforms to the "exactly -1" constraint varies with the sample. When including low-inflation countries, the model is more "loose"; while in purely high-inflation samples, although the direction is correct, the output-suppressing effect is significantly stronger than theoretically expected.

Practical Recommendations: For policymakers, in a high-inflation environment, controlling monetary growth is the only effective means of suppressing prices, while promoting economic growth can provide a stronger pressure-reducing effect than theoretically expected.