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Money Growth, Output Growth, and Inflation: Estimation of a Modern Quantity Theory

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This paper develops a long-run version of the quantity theory of money growth, real GDP growth, and inflation. Inflation rates, averaged for the years 1980–1993, are computed for 81 countries. These cross-section inflation rates are explained almost entirely by average M2 growth rates. In countries marked by high money growth and inflation, the estimated coefficients of M2 growth are strikingly close to one, strongly confirming the quantity theory. By contrast, in countries with relatively low money growth and inflation, the estimated money growth coefficient is only 0.69; the quantity theory offers a less complete explanation of inflation. Money growth and GDP growth are nearly orthogonal, consistent with long-run monetary superneutrality. The quantity theory is a reliable model of inflation for most countries, but not for those experiencing slow long-run money growth.

"The central predictions of the quantity theory are that, in the long run, money growth should be neutral in its effects on the growth rate of production and should affect the inflation rate on a one-for-one basis."
(Lucas 1996, p. 665)

"Inflation is always and everywhere a monetary phenomenon, produced in the first instance by an unduly rapid growth in the quantity of money."
(Friedman 1968, p. 18)

1. Introduction

During the years 1980–1993, annual inflation averaged 81.5% in Latin America, 12.3% in Africa, but only 6.4% among 16 countries in the OECD. Within Latin America, inflation ranged from 8.2% in Honduras to 374.3% in Argentina and in Africa from −0.6% in the Republic of the Congo to 61.6% in Sierra Leone. What are the reasons for these large differences? If cross-country differences in inflation can be linked closely to corresponding differences in money growth, the reason is clear. But if the link is weak, then other reasons must be sought.

Several authors have recently rejected money growth as an explanation of inflation. There are three lines of criticism. The first, typified by Baba, Hendry, and Starr (1992); Estrella and Mishkin (1997); and Cochrane (1998), argues that the income velocity of (and thus the demand for) monetary aggregates is so unstable that money growth is an unreliable explanation.

A second criticism, closely related to the first, is a time-series issue: If money, the price level, and output are not cointegrated, then there is no stable long-run relationship among them. Some

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researchers find evidence of cointegration (Hoffman and Rasche 1991; Baba, Hendry, and Starr 1992; Stock and Watson 1993; Hoffman, Rasche, and Tieslau 1995; Swanson 1998; Carlson et al. 2000; Dutkowsky and Atesoglu 2001), but others do not (Stock and Watson 1989; Hafer and Jansen 1991; Friedman and Kuttner 1993; Thoma 1994). The cointegration question remains unsettled.

A third criticism is quite distinct from the first two. It argues that in dynamic general equilibrium models with infinitely lived households, money in the household utility function, and rational expectations, there is a class of policy rules with a unique solution that shows that the price level is independent of monetary policy but dependent strictly on fiscal policy. This "fiscal theory of price level determination" *breaks any link between money growth and inflation.* The key fiscalist models were developed by Leeper (1991), Sims (1994, 1996), and Woodford (1994, 1995, 1998). A simple version is summarized succinctly by McCallum (2001), who demonstrates an alternative mathematical solution to a "fiscalist model" that yields a monetary explanation of the price level. But the monetarist foundations are under vigorous attack.[1]

This paper specifies and estimates a modern version of the quantity theory of money growth, real GDP growth, and inflation. Its traditional feature is that a country's long-run inflation rate increases with its money growth rate. The modern wrinkle is that inflation is mitigated by real GDP growth. I assume that real GDP growth is governed by exogenous forces such as growth in human capital, physical capital formation, and technological progress. The model makes no attempt to explain GDP growth rates; long-run neutrality is presupposed.

I show that the quantity theory can be written as a regression model whose theoretical implications can easily be tested (section 3). This paper then makes three empirical contributions. The first is to estimate a long-run version of the quantity theory and test its implications statistically (sections 5 and 6). The estimates are long run in the important sense that inflation, money growth, and real GDP growth rates are annual averages computed for 81 countries over the 14 years from 1980 to 1993. Short-run changes in the series are excluded by design. The second is to partition statistically the roles of money and real GDP growth as determinants of inflation (section 7). The third is to conduct out-of-sample forecasts (section 8). Here we discover how well the model, estimated for one group of countries, predicts inflation in others.

This paper differs from previous work in three important ways. First, earlier research has not been based on theoretically grounded regression models (Friedman 1956, 1968; Klein 1956; Friedman and Friedman 1980; Friedman and Schwartz 1982; McCandless and Weber 1995; Rolnick and Weber 1997). I advance this work by specifying the quantity theory as a regression model whose theoretical implications are tested explicitly.

Second, this paper develops a simple method to distinguish between money growth and real GDP growth as determinants of inflation. Both are statistically significant, but money growth is far more important.

Third, I estimate the model using subsamples of countries, then predict inflation outside the sample. The quantity theory predicts inflation with stunning accuracy for all countries experiencing actual inflation greater than 60% but less accurately for a group of 16 OECD countries characterized by low money growth. These results strongly suggest that high long-run inflation is driven by equally high long-term money growth. The relation is essentially one for one. But to forecast inflation within an important group of countries marked by low long-run money growth, the one-for-one relation breaks down.

[1] The fiscal theory of price level determination is vehemently criticized by Buiter (1999).

2. Background

The quantity theory can be traced to Richard Cantillon and David Hume. As Lucas (1996, p. 662) puts it, "These are two of Hume's statements of what we now call the Quantity Theory of Money: the doctrine that changes in the number of units of money in circulation will have proportional effects on all prices that are stated in money terms, and no effect at all on anything real, on how much people work or on the goods they produce or consume."

The quantity theory thus contains two testable propositions. The first is that long-run inflation rates are equal to money growth rates. The second is the long-run superneutrality of money: A country's long-run rate of real economic growth is independent of its money growth rate.[2]

Long-run superneutrality is of course not short-run superneutrality. Bruno and Easterly (1998) showed that countries experiencing short periods of high inflation also experienced decreases in growth of real GDP per capita. But following the episodes of high inflation, their per capita growth rates increased to rates above the world average (1998, tables 2 and 3). Money growth is far from neutral in the short run.[3]

Time-Series Evidence on Money Growth and Inflation

Cagan (1956) established the high correlation between monthly inflation and money growth during hyperinflations in Germany, Greece, Hungary, Poland, and Russia. Klein (1956), Friedman and Friedman (1980), and Lucas (1980) showed that annual inflation is closely correlated with annual money growth in Germany (Klein), in the United States (Lucas), and in United States, Germany, Japan, the United Kingdom, and Brazil (Friedman and Friedman). Hallman, Porter, and Small (1991) studied the dynamics of quarterly adjustment in inflation rates, adjustment of M2 velocity to its long-run equilibrium level (what they call the velocity gap), and adjustment of GNP to its long-run equilibrium level (what they call the output gap) in the United States for the years 1955–1988. Using inflation and monetary growth rates over periods averaging four years in the

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[2] Superneutrality need not hold even in the long run. If money is viewed as an alternative portfolio asset relative to real physical capital, then in an economy saving a constant proportion of disposable real income and a constant money growth rate, the long-run real capital-to-labor ratio will be lower in a monetary economy than in an economy without money. This class of monetary growth models was introduced by Tobin (1965) and analyzed further by Sidrauski (1967), Levhari and Patinkin (1968), and Stein (1969). In Tobin's model, monetary expansion is not superneutral for this reason: An increase in the rate of money growth raises the opportunity cost of holding assets as money. This higher opportunity cost of holding money induces households and firms to increase their rates of real capital accumulation, so that higher monetary expansion increases the rate of real capital accumulation until a new steady state with permanently higher capital intensity is attained. See also Tobin (1980). Walsh (1998) provides a detailed analysis of Tobin's (nonsuperneutral) model and a distinctly different model relying on money in the utility function, which results in superneutrality. Ahmed and Rogers (2000) review the steady-state properties of these two models as well as two cash-in-advance models that cannot imply superneutrality. Their careful empirical analysis of aggregate U.S. data indicates that a permanent unanticipated rise in inflation increases the long-run private investment rate, output, and consumption—outcomes consistent with Tobin's (1965) model.

[3] Time-series evidence roughly consistent with long-run superneutrality in the United Kingdom and the United States appears in Friedman and Schwartz (1982). As they put it, "For the United Kingdom, there seems little if any relation between monetary change and output: a simple quantity theory that regards price change as determined primarily by monetary change and output by independent other factors fits the evidence for the period as a whole (excluding wars, which we have largely omitted from our analysis" (p. 463). Cross-section independence of real GNP growth rates from money growth rates is shown for 62 countries by Dwyer and Hafer (1988) and for 110 countries by McCandless and Weber (1995). I find the correlation coefficient between money growth and real GDP growth for 29 low-income countries to be .275; for 29 upper-middle-income and high-income countries, it is −.219; and for the 17 high-income countries, it is .292. These low correlations hint at long-run superneutrality but are far from conclusive evidence.

United States and 5.6 years in the United Kingdom, Friedman and Schwartz (1982) found inflation to depend almost entirely on money growth.

Cross-Section Evidence

Others have explained differences in inflation rates across countries by differences in their money growth rates. The research setup has two steps. First, compute long-run average annual inflation and money growth rates for several countries. Second, estimate a cross-section regression of the countries' long-run inflation on their long-run money growth rates.

Lothian (1985) applied this method using 14-year average inflation and money (M1) growth rates for 20 OECD countries. He obtained a regression coefficient of .891 with standard error .149. Duck (1993) regressed 13-year average annual inflation on money (M2) growth rates using a 33-country sample. Like Lothian, he obtained regression coefficients close to one.

Using a 62-country sample, Dwyer and Hafer (1988) found a money growth coefficient of 1.031 with standard error .025. McCandless and Weber (1995) and Rolnick and Weber (1997) also found close cross-country correlation between long-term inflation and money growth.

These cross-section studies support a monetary explanation of long-run inflation. But they share a common statistical shortcoming: None tests for homoscedasticity or normality of regression disturbances. If the residuals were to display heteroscedasticity, the models should be reestimated with corrections for heteroscedastic errors. And standard t-tests applied to estimated regression coefficients also require normally distributed disturbances. It is thus worthwhile to report tests for homoscedasticity and normality in the regressions that follow.

3. The Quantity Theory as a Regression Model

Begin with the simplest quantity theory:

$$M_i^s V_i = P_i Q_i^D \text{ is a monetary measure of aggregate demand} \tag{1}$$

where M_i^s is the money supply (M2) in country i, V_i is the velocity of circulation in country i, P_i is the aggregate price level (GDP deflator) in country i, and Q_i^D is real GDP demanded in country i.

Taking logarithms of Equation 1 and differentiating with respect to time, we have

$$d \log P_i/dt = d \log M_i^s/dt + d \log V_i/dt - d \log Q_i^D/dt. \tag{2}$$

As it stands, Equation 2 provides no basis for statistical estimation. To express it as a regression equation, I make five simplifying assumptions:

(i) $d \log V_i/dt$ is random variable within each country that is uncorrelated with money growth and GDP growth.
(ii) $M_i^s = M_i^D$ (money supply = money demand).
(iii) M_i^s is exogenous.[4]
(iv) $Q_i^s = Q_i^D$ (aggregate real supply equals aggregate real demand).
(v) $d \log Q_i^s/dt$ is exogenous (long-run superneutrality).

[4] The money stock would be endogenous if the monetary authority were to employ interest rates as its primary policy instrument. For a review of earlier debates on exogeneity of the money stock, see Walsh (1998). In this single-equation model, it is convenient to view money as exogenous.

Using these five assumptions in Equation 2, one obtains

$$d \log P_i/dt = d \log M_i^s/dt - d \log Q_i^s/dt + u_i. \tag{3}$$

The disturbance term, u_i, is attributable chiefly to two sources. The first is errors in measuring inflation rates, which are likely to be larger in countries with very high inflation than in others.[5] The second is assumption (i) that within-country change in velocity is a random variable uncorrelated with money and real GDP growth. To view velocity changes as part of the disturbance term seems reasonable enough. But if these changes were cross-sectionally correlated with money growth, the estimated money growth coefficient would be biased.

Equation 3 can be written as a restricted regression model:

$$Y_i = X_i - Z_i + u_i, \tag{4}$$

where Y_i is the country's inflation rate, X_i its money growth rate, and Z_i its real GDP growth rate. The restrictions are that the intercept is zero, the coefficient of money growth is plus one, and the coefficient of real GDP growth is minus one.

Framed as an unrestricted regression, the quantity theory becomes

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 Z_i + \varepsilon_i, \tag{5}$$

where ε_i is a disturbance, *conditionally assumed* to be normally distributed with $E(\varepsilon_i) = 0$, $E(\varepsilon_i \varepsilon_j) = 0$ for $i \neq j$ (independent across countries) and $E(\varepsilon_i^2) = \sigma^2$ for $i = j$ (homoscedastic). The assumptions of normality and homoscedasticity are tested in all regressions that follow.[6]

Several hypotheses can be tested with Equation 5. The most stringent is that Equations 4 and 5 are equivalent: $\beta_0 = 0$, $\beta_1 = +1$, $\beta_2 = -1$; money growth increases, and GDP growth reduces, inflation one for one. Another is that the coefficients β_1 and β_2 are stable across different groups of countries. Why might they differ? One group of countries having monetary institutions quite different from another might have a different money growth coefficient. The quantity theory implies that β_1 and β_2 should be the same—that institutions should not matter—but reality might prove otherwise. Tests of coefficient stability are reported in section 5.

Third, the quantity theory implies that $\beta_1 = 1$ regardless of inflation. But β_1 might be smaller in countries with low inflation than in others. Why? Because M2 includes a much broader range of assets than M1. In countries experiencing hyperinflation and negative real interest rates, citizens have an incentive to hold most M2 assets as M1 to be spent promptly to minimize the inflation tax. If *real* interest rates on nontransaction assets were always positive (nominal rates adjust immediately to expected rates of inflation), however, the inflation tax would be minimized by holding the *minimum* feasible portion of M2 as M1 required for transactions.

In countries with little or no inflation and always positive real interest rates, people should be content to hold a larger percentage of M2 as nontransaction assets. If so, then low-inflation

[5] Error in measuring inflation, if it could be viewed in isolation, would not be a problem because the measurement error would simply be a component of the regression disturbance term. But, in fact, errors in measuring inflation transmit errors to measuring real GDP growth rates. Each country's real GDP growth rate is estimated by subtracting its estimated rate of inflation from its estimated nominal GDP growth rate. Assume that nominal GDP growth is correctly measured. If its estimated rate of inflation is overstated, its estimated real GDP growth rate must be understated. More generally, errors in measured inflation are negatively correlated with errors in measured real GDP growth. And measurement error in GDP growth creates a downward bias in its estimated coefficient regardless of sample size. One option would be to employ an instrumental variable for GDP growth, but I have not pursued it.

[6] When the assumptions of normality and homoscedasticity are tested in sections 4 and 5, they are sometimes satisfied and at other times violated.

countries would have a smaller β_1 than those with rampant inflation. In low-inflation countries, M2 growth could still drive inflation, but not one for one.

4. The Cross-Section Sample

The sample includes all countries for which data are published covering (i) average annual inflation rates, (ii) average annual rates of money growth, and (iii) average annual rates of real GDP growth for the period 1980–1993. The World Bank published these data for 81 countries.[7] Inflation rates are average percentage changes in each country's implicit GDP deflator.[8] Money growth rates are International Monetary Fund (IMF) estimates of each country's rate of M2 growth, where M2 is M1 plus time and savings deposits that can be converted to M1 with little delay or penalty (World Bank 1995). M2 is defined consistently across countries by the IMF and is the preferred measure of each country's money supply.

The IMF does not publish money growth rates for the 15 republics of the former Soviet Union for two reasons. First, most are not members of the IMF. Second, those that recently joined have very short monetary histories. Nor does the IMF publish money growth rates for countries that during this period experienced major currency reforms (such as Brazil) or for countries whose M2 cannot reliably be estimated.

I should mention that average annual inflation and real GDP growth rates covering 131 countries for the years 1990–1998 were recently published by the World Bank (2000). But the IMF has not published average annual M2 growth rates for this period. Year-to-year changes published by the IMF are often quite erratic and in many countries frequently negative. I was unsuccessful in attempting to use these numbers to obtain meaningful long-run averages. If the IMF were to publish long-run M2 growth rates that could be matched with corresponding inflation and real GDP growth rates, the sample used here could be updated and the regressions reestimated.

But the 81-country sample is globally representative, covering more than 90% of the world's population. It includes 27 African countries, 15 Latin American countries, China, India, Pakistan, and Indonesia, and 16 OECD countries.

Tests based on diverse cross-country samples offer three advantages. First, these countries have monetary systems that differ widely: Some are highly sophisticated, as those in the OECD countries, and others are in formative stages, as those in the Gambia, Niger, or Togo. Second, some countries have monetary authorities largely independent from their central governments, such as the United States. In others, the monetary authority and central government are one and the same (e.g., Argentina, Venezuela, Bolivia, Ecuador, China, and Korea). Third, the countries exhibit widely varying inflation rates during the period 1980–1993, ranging from 374.3% in Argentina to −0.6% in the Republic of the Congo. A theory that could satisfactorily explain such enormous differences would be quite general.

5. Full Sample Estimates

I first estimate Equation 5 by applying ordinary least squares (OLS) to the 81-country sample. Results are shown in Table 1, panel a. The model fits the data extremely well, with an adjusted R^2

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[7] These data are available from the author on request.

[8] Although subject to some limitations, percentage changes in implicit GDP deflators are the broadest measure of domestic inflation. See World Bank (1995).

Table 1. Estimates of Equation 5, 81 Countries

(a) OLS estimates		
Money Growth Coefficient		Real GDP Growth Coefficient
$\hat{\beta}_1 = 1.033$		$\hat{\beta}_2 = -1.748$
Standard error = .009		Standard error = .228
	Mean inflation rate = 25.10%	
	Residual variance = 18.72	
	$\overline{R}^2 = .995$	
(b) Heteroscedasticity-corrected estimates (all variables divided by $\sqrt{X_i}$)		
Money Growth Coefficient		Real GDP Growth Coefficient
$\hat{\beta}_1^* = 1.011$		$\hat{\beta}_2^* = -1.551$
Standard error = .020		Standard error = .138
	$\overline{R}^2 = .954$	

= .995. The regression intercept is not significantly different from zero. The money growth coefficient of 1.033 is strikingly close to one, consistent with the quantity theory. Because its estimated standard error is so small, $\hat{\beta}_1$ appears to be significantly larger than one at $p \leq .01$ (the t-statistic is .033/.0090 = 3.67, and the critical t to reject $H_0{:}\beta_1 = 1$ is 2.65). However, heteroscedastic and nonnormal regression residuals make this test inexact, as discussed later.

The real GDP growth coefficient, -1.748, appears to be significantly smaller than the quantity theory restriction $\beta_2 = -1$ (the t-statistic for a test of $\beta_2 = -1$ is $-748/.228 = -3.28$). This test, too, is questionable because of heteroscedastic, nonnormal residuals.

Not surprisingly, White's test for homoscedasticity shows that the squared residuals are much larger for countries with higher money growth and leads to rejection of the hypothesis of homoscedastic disturbances. Likewise, the Jarque–Bera test shows that the residuals are highly skewed, so normality must be rejected.[9] Tests of significance based on these OLS estimates are therefore inexact.

To correct for heteroscedasticity, I assume $\sigma_i^2 = \sigma^2 X_i$, (variances of the original disturbances are proportional to X_i) and divide all observations by $\sqrt{X_i}$. Equation 5 is reestimated with results shown in Table 1, panel b. The estimated money growth coefficient $\hat{\beta}_1^*$ is highly significant, but at $p \leq .01$ is not significantly different from the quantity theory hypothesis $\beta_1 = 1$. But the estimated real GDP growth coefficient $\hat{\beta}_2^* = -1.551$ is significantly different from the quantity theory hypothesis $\beta_2 = -1$.[10]

[9] The test statistic using White's test for homoscedasticity is asymptotically distributed as chi-squared with $m - 1$ degrees of freedom, where m is the number of coefficients (excluding the constant term) in a regression of the squared residuals on explanatory variables and squared values of explanatory variables used in the initial OLS regression. The computed test statistic is 28.5, so homoscedasticity is rejected at $p \leq .01$. The Jarque–Bera test for normality is based on coefficients of skewness and kurtosis obtained from the OLS residuals. If a random variable is precisely normally distributed, skewness is zero, the kurtosis coefficient minus 3 is zero, and the Jarque–Bera statistic is zero. This statistic is asymptotically distributed as chi-squared with two degrees of freedom. The critical value of chi-squared to reject normality at $p \leq .05$ is 5.99 and at $p \leq .01$ is 9.21. A calculated Jarque–Bera statistic larger than 9.21 almost surely indicates nonnormal regression disturbances; hence, the estimated regression coefficients are not normally distributed, so t-tests and F-tests based on these estimated coefficients are inexact at the stipulated level of significance. The calculated Jarque–Bera statistic obtained from the residuals of the OLS regression in Table 1, panel a, is 659.

[10] Residuals obtained after this correction for heteroscedasticity displayed no evidence of heteroscedasticity or nonnormality. White's test of the null hypothesis of homoscedasticity has a test statistic $nR^2 = 3.98$; the computed Jarque–Bera statistic for testing normality is 2.41.

To summarize: All three estimates of β_1 are very close to one, and the heteroscedasticity-corrected estimates of β_1 are not statistically different from one. In fact, using the heteroscedasticity-corrected estimates, the joint hypotheses $\beta_0 = 0$ and $\beta_1 = 1$ and $\beta_2 = -1$ cannot be rejected at $p \leq .01$. The quantity theory hypotheses $\beta_1 = 1$ and $\beta_2 = -1$ are consistent with the 81-country sample evidence.

6. Tests of Coefficient Stability

Does the quantity theory apply equally well to different groups of countries? One might imagine that the theory fits the inflationary experiences of countries with highly integrated commercial banks and nonbank financial institutions but fails to apply in countries with less sophisticated financial systems. The 81-country sample can be divided in many ways. The results of what seem to be five fairly natural divisions are reported here.

Test 1: Low-Income (Group 1) versus Upper-Middle and Upper-Income (Group 2) Countries

The World Bank makes a convenient division of countries as low-income, lower-middle-income, upper-middle-income, and high-income economies. The 29 lowest-income economies in the sample (Group 1) have 1993 per capita GNP ranging from 750 to 3,150 international dollars. The 29 upper-middle and high-income economies in the sample (Group 2) exhibit per capita GNP ranging from 8,130 to 24,740 international dollars.[11]

Equation 5 is estimated for the two groups, with results in Table 2. The regression clearly fits well in both groups. The estimated money growth coefficients are nearly equal; neither is significantly different from one. Assuming the samples are independent, one may test the null hypothesis $\beta_{1(\text{Group 1})} = \beta_{1(\text{Group 2})}$. The test-statistic has a t-distribution. The calculated t is 0.895, so one cannot reject the hypothesis of equal coefficients. Again, money growth appears to drive inflation one for one.

The estimated coefficients of real GDP growth are also quite similar: -1.67 for Group 1 and -1.79 for Group 2. Computing

$$[\hat{\beta}_{2(\text{Group 1})} - \hat{\beta}_{2(\text{Group 2})}]/\text{standard error of } [\hat{\beta}_{2(\text{Group 1})} - \hat{\beta}_{2(\text{Group 2})}]$$

yields a calculated t-statistic of $.122/.345 = .356$, so we accept the null hypothesis of equal coefficients.[12]

[11] The World Bank estimates of GNP per capita are based on purchasing power parity adjustments used in the International Comparison Project (ICP). By expressing the estimates in international dollars, an attempt is made to equalize price levels across countries, thereby yielding estimates of GNP per capita that are internationally comparable. Although these estimates have serious limitations, they are the most closely comparable estimates of real GNP per capita currently available. Data are obtained from World Bank (1995, table 30).

[12] The Jarque–Bera test shows that normality of regression residuals cannot be rejected in either group. The computed Jarque–Bera statistic for Group 1 is 7.34, and that for Group 2 is .815. The critical value to reject normality at $p \leq .01$ is 9.21. White's statistic for testing homoscedasticity in Group 1 is 6.75, and in Group 2 it is 11.50, indicating that homoscedasticity cannot be rejected at $p \leq .01$.

Table 2. Estimates of Equation 5, 29 Low-Income Countries (Group 1) and 29 Upper–Middle and High-Income Countries (Group 2)

	Group 1	
Money Growth Coefficient $\hat{\beta}_1 = 1.093$ Standard error = .050	Mean inflation rate = 13.00% Residual variance $s_1^2 = 12.48$ $\overline{R}^2 = .945$	Real GDP Growth Coefficient $\hat{\beta}_2 = -1.670$ Standard error = .375
	Group 2	
Money Growth Coefficient $\hat{\beta}_1 = 1.048$ Standard error = .007	Mean inflation rate = 25.86% Residual variance $s_2^2 = 5.23$ $\overline{R}^2 = .999$	Real GDP Growth Coefficient $\hat{\beta}_2 = -1.792$ Standard error = .209

Test 2: Africa Compared with Latin America

Equation 5 is next estimated for two economically distinct groups of countries. The first consists of 27 African countries and the second 15 Latin American nations. Both samples exhibit wide ranges of money growth and inflation. Inflation in African countries ranged from 61.6% in Sierra Leone to −0.6% in the Republic of the Congo, with mean inflation of 12.3%. But in Latin America, inflation ranged from 374.3% in Argentina to 8.2% in Honduras, with a mean rate of 81.5%.

Estimates of Equation 5 are in Table 3. The estimated money growth coefficients for the African and Latin American countries are nearly equal, and neither appears to be significantly different from one at $p \leqslant .01$. A test of the null hypothesis that the coefficients are equal yields a calculated t of 1.07; the hypothesis of equal coefficients obviously cannot be rejected at $p \leqslant .01$.

The real GDP growth coefficients have a wider gap: $\hat{\beta}_2$ for the African sample is −1.297, while $\hat{\beta}_2$ for the Latin American sample is −2.122. Because the Latin American real GDP growth coefficient is estimated imprecisely, the two estimates are not significantly different from each other. The large estimated standard error of $\hat{\beta}_2$ for the Latin American sample seems linked to the fact that these countries showed little dispersion in GDP growth. This sample also includes several countries with extremely high inflation, which could cause measurement errors in their reported estimates of GDP growth. But the quantity theory regression fits the African and the Latin American samples reasonably well.[13]

Test 3: High-Income Countries

The World Bank classified 24 countries in 1995 as "high income economies."[14] My sample includes 17 of them.[15] These economies have highly sophisticated money and credit markets.

[13] Residuals from the regression applied to African countries appear to be normally distributed: The Jarque–Bera statistic is 0.496. But the residuals from Latin America decisively fail the test for normality: They are negatively skewed and leptokurtic. The Jarque–Bera statistic of 21.83 far exceeds the critical Π^2 value of 9.21 at $p \leqslant .01$.

[14] High-income countries are those with real purchasing power parity adjusted GNP per capita in 1993 of at least $13,490. See World Bank (1995).

[15] It excludes New Zealand, Hong Kong, United Kingdom, Kuwait, the Netherlands, Belgium, and the United Arab Emirates because either inflation rates or M2 growth rates are not published.

Table 3. Estimates of Equation 5

	(a) 27 African countries	
Money Growth Coefficient		Real GDP Growth Coefficient
$\hat{\beta}_1 = 1.088$		$\hat{\beta}_2 = -1.297$
Standard error = .0411		Standard error = .294
	Mean inflation rate = 12.34%	
	Residual variance = 8.91	
	$\overline{R}^2 = .968$	
	(b) 15 Latin American countries	
Money Growth Coefficient		Real GDP Growth Coefficient
$\hat{\beta}_1 = 1.035$		$\hat{\beta}_2 = -2.122$
Standard error = .0279		Standard error = 2.354
	Mean inflation rate = 81.53%	
	Residual variance = 71.89	
	$\overline{R}^2 = .995$	

Apart from Israel (with average annual inflation of 70%), all experienced moderate inflation: Their mean inflation rate was 9.33%.

Results from estimating Equation 5 are shown in Table 4. The money growth coefficient of .968 is estimated precisely but is not statistically different from the quantity theory hypothesis $\beta_1 = 1$. The estimated real GDP growth coefficient is not significantly different from the quantity theory hypothesis $\beta_2 = -1$. A test of the joint restrictions $\beta_1 = 1$ and $\beta_2 = -1$ yields an F-statistic of 2.68, so the combined restrictions of the quantity theory cannot be rejected at $p \leq .05$. The quantity theory closely fits the inflationary, monetary and real GDP growth experience of these countries. The regression residuals show no evidence of heteroscedasticity and no departure from normality.[16]

Two Further Tests

The heart of the quantity theory is that money growth drives inflation one for one, always and everywhere. If true, then strong tests can be developed by estimating the model for two groups of countries distinguished in one important way. The first group must display large money growth relative to the growth of output; money growth should be the primary determinant of inflation, with real GDP growth being of minimal importance. The second group should feature much lower growth of money relative to output; money and output growth would be put on more nearly equal footing.[17]

Thirty-six countries displayed money growth exceeding GDP growth by at least 15%. The mean money growth rate is 53.3%, mean GDP growth is 2.7%, and mean inflation is 50.1%. Twenty-one countries showed money growth exceeding GDP growth by less than 6%. In this group, mean money growth is only 5.9%, mean GDP 2.4%, and mean inflation 2.9%. It is convenient to view the first group as "high-money-growth, high-inflation" and the second as "low-money-growth, low-inflation" countries.

[16] White's test statistic is 3.66, consistent with the hypothesis of homoscedastic residuals. The Jarque–Bera test for normality yields a calculated value of 0.518, so the residuals are practically normally distributed.

[17] In an earlier version of this paper I split the sample according to countries experiencing average annual inflation of at least 15% and those with inflation of 8.5% or less. A referee suggested instead that the sample be divided according to differences in growth rates of money and real GDP. In the earlier version, the quantity theory regression afforded low explanatory power in the "low-inflation" sample. And the same is true for the quantity theory regression reported here in Table 6.

Table 4. Estimates of Equation 5, 17 High-Income Countries

Money Growth Coefficient		Real GDP Growth Coefficient
$\hat{\beta}_1 = .968$		$\hat{\beta}_2 = -1.479$
Standard error = .0262		Standard error = .360
	Mean inflation rate = 9.33%	
	Residual variance = 2.92	
	$\bar{R}^2 = .989$	

Test 4: High-Money-Growth, High-Inflation Countries

These countries include 10 African nations, 13 Latin American nations, plus China, Egypt, Myanmar, Indonesia, the Philippines, Romania, Poland, Turkey, Iran, Mauritius, Greece, Portugal, and Israel. Money growth ranges from 16.6% to 356.7%.

Applying OLS to Equation 5 yields results reported in Table 5, panel a. The estimated money growth coefficient is estimated with high statistical precision but appears to be not statistically different from one at $p \leqslant .01$. The GDP growth coefficient is also estimated with high precision. Since $\bar{R}^2 = .994$, the regression fits almost perfectly. Despite these impressive facts, the OLS regression residuals are not distributed normally and are probably heteroscedastic.[18] Accordingly, Equation 5 is reestimated using weighted least squares.

The original observations are divided by $\sqrt{X_i}$ (money growth rates), then Equation 5 is reestimated. As shown in Table 5, panel b, the money growth coefficient is 26 times its estimated standard error but is not significantly different from one. The GDP growth coefficient is also highly significant. Residuals obtained from this weighted-least-squares regression appear to be normally distributed and homoscedastic.[19] Most important, money growth and inflation are one for one.

Test 5: Low-Money-Growth, Low-Inflation Countries

The 21 countries with money growth exceeding real GDP growth by less than 6% include 11 African and eight OECD countries plus Papua New Guinea and Oman. These countries could scarcely be more diverse geographically, culturally, or economically. Their only common thread is relatively low money growth, ranging from 2.5% to 9.6%. And low money growth turns out to be the key to a poor performance by the quantity theory.

Equation 5 yields estimates reported in Table 6. One contrast with previous estimates is clear. The estimated money growth coefficient is 0.694. This estimate is not statistically significant at $p \leqslant .01$ (but is significant at $p \leqslant .05$). Despite the fact that money growth coefficient is not statistically different from one, the one-for-one relationship found in earlier estimates seems tenuous in low-inflation countries.

The GDP growth coefficient is estimated somewhat more precisely (3.3 times its estimated standard error). It is highly significant but not different from the quantity theory hypothesis $\beta_2 = -1$.

Another contrast with the preceding estimates is clear: $\bar{R}^2$ is only .308, less than one-third the values reported in preceding tables. Inflation in these countries leaves much to be explained by factors other than money and GDP growth. In this environment of low money growth and low

[18] The calculated Jarque–Bera statistic is 117.9, indicating highly skewed residuals. White's test statistic for testing homoscedasticity is 14.13, exceeding the critical value to reject homoscedasticity at $p \leqslant .01$.

[19] The computed Jarque–Bera statistic is 1.83. White's statistic for testing homoscedasticity, $nR^2 = 3.22$, shows no evidence of heteroscedastic residuals.

Table 5. Estimates of Equation 5, 36 High-Money-Growth, High-Inflation Countries

(a) OLS estimates

Money Growth Coefficient		Real GDP Growth Coefficient
$\hat{\beta}_1 = 1.034$		$\hat{\beta}_2 = -2.053$
Standard error = .014		Standard error = .483
	Mean inflation rate = 50.1%	
	Residual variance = 36.17	
	$\bar{R}^2 = .994$	

(b) Heteroscedasticity-corrected estimates (all variables divided by $\sqrt{X_i}$)

Money Growth Coefficient	Real GDP Growth Coefficient
$\hat{\beta}_1^* = 1.014$	$\hat{\beta}_2^* = -1.981$
Standard error = .039	Standard error = .314
$\bar{R}^2 = .966$	

inflation, the quantity theory seems inadequate. Despite the low $\bar{R}^2$, analysis of the residuals indicates that the regression is technically well specified.[20]

7. Partitioning the Sources of Inflation: Money Growth and Real GDP Growth

Other things equal, more rapid money growth stimulates inflation, and real GDP growth mitigates it. But what is the relative importance of these two opposing forces? With Equation 5, one may easily quantify the relative impacts of money growth and GDP growth on inflation.

Consider the complete 81-country sample. The variance in inflation rates explained by Equation 5 can be partitioned as

$$M_{\hat{y}\hat{y}} = \underbrace{\hat{\beta}_1^2 M_{xx}}_{\substack{\text{explained by}\\ \text{money growth}}} + \underbrace{\hat{\beta}_2^2 M_{zz}}_{\substack{\text{plained by}\\ \text{real GDP growth}}} + \underbrace{2\hat{\beta}_1\hat{\beta}_2 M_{xz}}_{\substack{\text{explained jointly by}\\ \text{money growth and}\\ \text{real GDP growth,}}} \qquad (6)$$

where M_{xx} and M_{zz} are the sample variances in X and Z and M_{xz} is the sample covariance between X and Z. The covariance between X and Z is quite small, so the influences of money and real GDP growth are nearly separate and additive.[21] Substituting the regression coefficients from Table 1, panel a, and the sample variances $M_{xx} = 2947$, $M_{zz} = 5.046$, and covariance $M_{xz} = -22.89$, the explained regression variance is

$$M_{\hat{y}\hat{y}} = \underbrace{3145}_{\substack{\text{explained by}\\ \text{money growth}}} + \underbrace{15.42}_{\substack{\text{explained by}\\ \text{real GDP growth}}} + \underbrace{82.66}_{\substack{\text{explained jointly by}\\ \text{money growth and}\\ \text{real GDP growth}}} = 3243.08.$$

[20] The Jarque–Bera statistic is 1.41, and White's test statistic is 4.99.

[21] The correlation coefficient between money growth and real GDP growth is $r_{xz} = -.195$. If r_{xz} were zero, money growth and real GDP growth would be orthogonal and their contributions to explained variance strictly additive.

Table 6. Estimates Equation 5, 21 Low-Money-Growth, Low-Inflation Countries

Money Growth Coefficient		Real GDP Growth Coefficient
$\hat{\beta}_1 = .694$		$\hat{\beta}_2 = -1.072$
Standard error = .278		Standard error = .324
	Mean inflation rate = 2.88%	
	Residual variance = 3.15	
	$\overline{R}^2 = .308$	

The covariance of money growth and real GDP growth contributes almost nothing to explained regression variance, but the variance explained by money growth alone is enormous. Ignoring the covariance of 82.66, money growth alone contributes at least 3145/3243.08 = .970, or 97%, of explained regression variance. Since the overall model R^2 is .995, the large differences in inflation are explained *almost entirely by differences in money growth rates.*

Real GDP growth may be *statistically* significant, but if we ignore the 82.66 covariance term, the contribution of GDP growth has a lower bound of 15.42/3243.08 = .0048, or one-half of 1%.

Suppose instead that one credits *all* explanatory power in the covariance term to GDP growth. Then GDP growth has an upper bound to explained variance of (15.42 + 82.66)/3243.08 = 98.08/3243.08 = .0302, or 3%. The conclusion is inescapable: International variation in inflation is attributable almost entirely to corresponding variation in money growth rates. Money matters.

8. Out-of-Sample Predictions

That the quantity theory regression fits well within a sample of countries says nothing about its ability to predict inflation in others. To test its predictions, I conduct two experiments. In the first, I estimate the model using observations on the 74 countries with annual inflation rates lower than 60%. The estimated coefficients are then used to predict inflation in seven countries with annual inflation greater than 60%. Results are in Table 7. The regression closely predicts inflation in all countries, and with astonishing accuracy in Peru, Poland, Uruguay, and Argentina. Its predictive power is based on high money growth rates in all these countries.

In the second experiment, I estimate the model using the 65 non-OECD countries, then predict inflation in 16 OECD nations. Results are in Table 8. The regression predicts inflation with tolerable accuracy for most countries. The only serious error occurs in Ireland, where actual inflation averaged 4.8% but predicted inflation is less than 1%. But these predictions are only moderately successful, and for this reason: because the mean absolute prediction error of 1.5% is *relatively high* among these countries, in which actual inflation averaged only 6.4%.

Table 7. Actual and Predicted Inflation in Seven High-Inflation Countries, 1980–1993

Country	Annual Inflation (%)	Predicted Annual Inflation (%)	Prediction Error[a]
Sierra Leone	61.6	56.4	5.2
Israel	70.4	73.2	−2.8
Bolivia	187.1	209.0	−21.9
Peru	316.1	302.7	13.4
Poland	69.3	64.1	5.2
Uruguay	66.7	69.2	−2.5
Argentina	374.3	361.7	12.6

[a] Mean absolute prediction error = 9.1%.

Table 8. Actual and Predicted Inflation in 16 OECD Countries, 1980–1993

Country	Annual Inflation (%)	Predicted Annual Inflation (%)	Prediction Error[a]
Greece	17.3	20.6	−3.3
Portugal	16.4	13.7	2.7
Ireland	4.8	0.8	4.0
Spain	8.4	6.4	2.0
Australia	6.1	6.3	−0.2
Finland	5.8	8.1	−2.3
Italy	8.8	6.8	2.0
Canada	3.9	3.6	0.3
Austria	3.6	3.3	0.3
Germany	2.7	2.3	0.4
Sweden	6.9	3.9	3.0
United States	3.8	2.5	1.3
Norway	4.6	5.1	−0.5
Denmark	4.6	6.4	−1.8
Japan	1.5	1.3	0.2
Switzerland	3.8	2.9	0.9

[a] Mean absolute prediction error = 1.5%.

9. Conclusions

The quantity theory can be written as a conditional regression model with testable implications. The model is transparent. Its statistical properties are easily analyzed and its implications easily tested. These are major strengths.

My tests confirm its major prediction that high inflation is a monetary phenomenon. High inflation is driven by rapid money growth, and the relation is essentially one for one. Applied to the full sample (Table 1), the Latin American sample (Table 3), the high-income sample (Table 4), and the high-money-growth, high-inflation sample (Table 5), the estimated money growth coefficients are remarkably close to one, and the GDP growth coefficients are often not statistically different from minus one.

But the quantity theory is far less successful in explaining inflation in countries characterized by low money growth. Applied to the 21 countries with mean annual money growth of 5.9% and mean annual inflation of 2.9%, the regression explains less than one-third of the total variance (Table 6). The money growth coefficient is estimated with relatively low statistical precision. The quantity theory affords an incomplete explanation of inflation. In the past decade, those who have rejected money growth and in its place have proposed a fiscal theory of the price level and inflation may realize this.

The quantity theory yields a rich set of testable propositions. Further tests with new cross-section samples would help in assessing its generality. Since the regression fits so poorly when applied to countries with low money growth, further work with such countries is an obvious priority for future research.

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