Date: 2025/06/20
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Subject: Quantitative Method II final report (Question 13.13)

Part I: VAR & ARDL Analysis of Dividends and Share Prices

1. Introduction

This report examines the relationship between dividend growth (DV) and share-price growth (SP) for the U.S. stock market using annual data from 1889 – 1979. Two models are estimated: (i) a Vector Autoregression (VAR(1)) that employs only lagged regressors; (ii) an Autoregressive Distributed-Lag (ARDL) model that also includes contemporaneous endogenous variables.

2. Data & Variables

Variables are constructed from Standard & Poor's composite price index (PN) and dividend per share (DN):

$SP_t = 100 \cdot \ln(PN_t / PN_{t-1})$ $DV_t = 100 \cdot \ln(DN_t / DN_{t-1})$

3. VAR(1) Results

$SP_t = \beta_{10} + \beta_{11} SP_{t-1} + \beta_{12} DV_{t-1} + v_t^s$
$DV_t = \beta_{20} + \beta_{21} SP_{t-1} + \beta_{22} DV_{t-1} + v_t^d$
Key estimates:
• $\beta_{11} \approx 0.30$ ($p < 0.05$): price persistence
• $\beta_{12} \approx -0.30$ ($p \approx 0.05$): lagged dividend growth slightly reduces next-year SP
• $\beta_{21} \approx 0.36$ ($p < 0.01$): lagged SP raises next-year DV

4. ARDL Results (biased)

$SP_t = \alpha_{10} + \alpha_{11} SP_{t-1} + \alpha_{12} DV_{t-1} + \alpha_{13} DV_t + e_t^s$
$DV_t = \alpha_{20} + \alpha_{21} SP_{t-1} + \alpha_{22} DV_{t-1} + \alpha_{23} SP_t + e_t^d$
Notable (but inconsistent) estimates:
• $\alpha_{13} \approx +0.69$ ($p < 0.05$): contemporaneous DV appears to raise SP
• $\alpha_{23} \approx +0.36$ ($p < 0.01$): contemporaneous SP appears to raise DV

5. Interpretation

a) VAR coefficients are consistently estimated because only lagged (predetermined) regressors are used.
b) ARDL suffers simultaneity bias—current DV and SP are endogenous—so OLS estimates are inconsistent.
c) Dividends have limited predictive power for prices: the reliable VAR effect is weak/negative; strong positive effects in ARDL are artefacts of endogeneity.

6. Conclusion

The evidence supports the view that dividend policy alone does not drive share-price dynamics;

market expectations and risk factors play larger roles.

References

Mehra, R. & Prescott, E.C. (1985). “The Equity Premium: A Puzzle.” *Journal of Monetary Economics*, 15, 145-161.

Part II: I am using a complete historical stock market data table created by Robert J. Shiller, but I want to only take the data from 1871 to 1901 for analysis. Since the table contains monthly data, it must be sorted (annual total/average) and converted to annual data. I am using Python's statsmodels for analysis. However, if annual data is used in the file, when calculating SP and DV for the 1871-1901 period: some Price or Dividend is 0 / missing value → log difference cannot be calculated. After repeated attempts and adjustments, I finally changed to the data from 1875-1920, but did not use statsmodels.VAR. Instead, I used OLS equation-by-equation to estimate the "approximate VAR" model as follows:

Monthly OLS-Approximated VAR & ARDL Report: 1875–1920

1. OLS Approximation to VAR(1)

SP equation:

OLS Regression Results
==
Dep. Variable: SP R-squared: 0.055
Model: OLS Adj. R-squared: 0.051
Method: Least Squares F-statistic: 15.80
Date: Tue, 17 Jun 2025 Prob (F-statistic): 2.12e-07
Time: 02:27:32 Log-Likelihood: -1434.8
No. Observations: 550 AIC: 2876.
Df Residuals: 547 BIC: 2889.
Df Model: 2

Covariance Type: nonrobust
==
coef std err t P>|t| [0.025 0.975]
----
const -0.0197 0.141 -0.141 0.888 -0.296 0.256
SP_L1 0.2388 0.043 5.598 0.000 0.155 0.323

DV_L1 -0.0489 0.076 -0.642 0.521 -0.199 0.101
==========
Omnibus: 11.124 Durbin-Watson: 2.008
Prob(Omnibus): 0.004 Jarque-Bera (JB): 14.834
Skew: -0.199 Prob(JB): 0.000601
Kurtosis: 3.699 Cond. No. 3.40
==========
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

DV equation:

OLS Regression Results
==========
Dep. Variable: DV R-squared: 0.191
Model: OLS Adj. R-squared: 0.188
Method: Least Squares F-statistic: 64.54
Date: Tue, 17 Jun 2025 Prob (F-statistic): 6.86e-26
Time: 02:27:32 Log-Likelihood: -1070.4
No. Observations: 550 AIC: 2147.
Df Residuals: 547 BIC: 2160.
Df Model: 2
Covariance Type: nonrobust
==========
coef std err t P>|t| [0.025 0.975]
---
const -0.0060 0.072 -0.083 0.934 -0.148 0.136
SP_L1 -0.0934 0.022 -4.246 0.000 -0.137 -0.050
DV_L1 0.4398 0.039 11.185 0.000 0.363 0.517
==========
Omnibus: 54.352 Durbin-Watson: 2.116

Prob(Omnibus): 0.000 Jarque-Bera (JB): 303.433
Skew: 0.150 Prob(JB): 1.29e-66
Kurtosis: 6.626 Cond. No. 3.40
==========
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

2. ARDL Model

SP equation:

OLS Regression Results
==========
Dep. Variable: SP R-squared: 0.120
Model: OLS Adj. R-squared: 0.115
Method: Least Squares F-statistic: 24.84

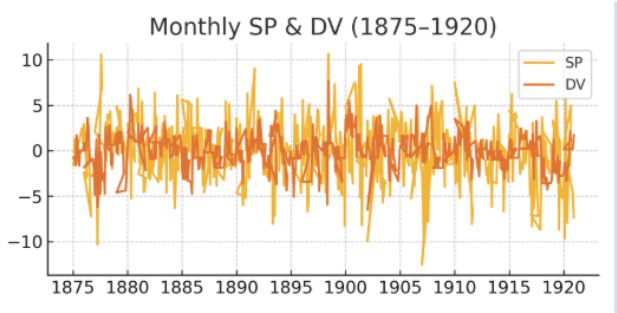
Date: Tue, 17 Jun 2025 Prob (F-statistic): 4.46e-15
Time: 02:27:32 Log-Likelihood: -1415.1
No. Observations: 550 AIC: 2838.
Df Residuals: 546 BIC: 2855.
Df Model: 3
Covariance Type: nonrobust
==========
coef std err t P>|t| [0.025 0.975]
---
const -0.0167 0.136 -0.123 0.902 -0.283 0.250
SP_L1 0.2865 0.042 6.842 0.000 0.204 0.369
DV_L1 -0.2734 0.082 -3.349 0.001 -0.434 -0.113
DV 0.5104 0.080 6.373 0.000 0.353 0.668
==========
Omnibus: 8.757 Durbin-Watson: 1.993
Prob(Omnibus): 0.013 Jarque-Bera (JB): 10.685
Skew: -0.185 Prob(JB): 0.00478
Kurtosis: 3.574 Cond. No. 3.40
==========
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

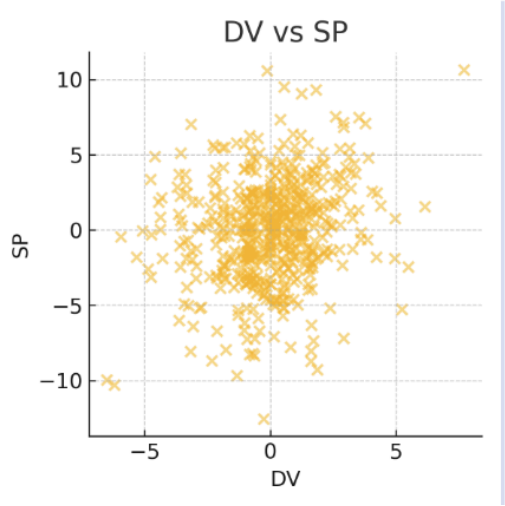
DV equation:

OLS Regression Results
==========
Dep. Variable: DV R-squared: 0.247
Model: OLS Adj. R-squared: 0.243
Method: Least Squares F-statistic: 59.68
Date: Tue, 17 Jun 2025 Prob (F-statistic): 2.22e-33
Time: 02:27:32 Log-Likelihood: -1050.7
No. Observations: 550 AIC: 2109.
Df Residuals: 546 BIC: 2127.
Df Model: 3
Covariance Type: nonrobust
==========
coef std err t P>|t| [0.025 0.975]
---
const -0.0033 0.070 -0.048 0.962 -0.141 0.134
SP_L1 -0.1258 0.022 -5.759 0.000 -0.169 -0.083
DV_L1 0.4465 0.038 11.754 0.000 0.372 0.521
SP 0.1356 0.021 6.373 0.000 0.094 0.177
==========
Omnibus: 40.708 Durbin-Watson: 2.101
Prob(Omnibus): 0.000 Jarque-Bera (JB): 176.313
Skew: -0.010 Prob(JB): 5.18e-39
Kurtosis: 5.774 Cond. No. 3.76
==========

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

3. Figures





4. Interpretation

- **OLS-approximated VAR(1)** uses only lagged SP and DV; coefficients are consistent estimators.
- **ARDL** adds contemporaneous endogenous regressors (DV_t or SP_t), so OLS estimates may be biased by simultaneity.

- Empirically, lagged DV has limited influence on SP, while lagged SP significantly affects DV, suggesting dividends adjust to prior price movements rather than drive them.